TM5 Pr 2.43

1) TM5 2-43 A PARTICLE ACTS UNDER $F = -kx + \frac{kx^3}{\alpha^2}$. DETERMINE $U(x)$ AND DISCUSS THE MOTION. WHAT HAPPENS WHEN $E = \frac{1}{4}k\alpha^2$?

FIND $U(x)$ BY INTEGRATION $U(x) = -\int \vec{F} \cdot d\hat{x}$

$$U(x) = \int \left(kx - \frac{kx^3}{\alpha^2}\right) dx = \boxed{\frac{1}{2}kx^2 - \frac{kx^4}{4\alpha^2} = U(x)}$$

TO SEE IF $E = \frac{1}{4}k\alpha^2$ IS IMPORTANT, FIND EQUILIBRIUM POINTS

$$\left.\frac{dU}{dx}\right|_{x_0} = \left. kx - \frac{kx^3}{\alpha^2}\right|_{x_0} = 0$$

$$\Rightarrow \cancel{k}x_0 = \frac{\cancel{k}x_0^{\cancel{3}\,2}}{\alpha^2}$$

$$\alpha^2 = x_0^2$$

$$\Rightarrow x_0 = \alpha$$

WHAT IS $U(x_0)$?

$$U(x_0) = \frac{1}{2}k\alpha^2 - \frac{k\alpha^{\cancel{4}\,2}}{4\cancel{\alpha^2}}$$

$$= \frac{1}{2}k\alpha^2 - \frac{1}{4}k\alpha^2 = \boxed{\frac{1}{4}k\alpha^2 = U(x_0 = \alpha)}$$

$\Rightarrow$ WHEN $E = \frac{1}{4}k\alpha^2$, IT'S EQUAL TO THE MAXIMUM $U(x)$

$\Rightarrow$ UNSTABLE EQUILIBRIUM POINT